Singapore Examinations and Assessment Board

Cambridge Assessment International Education

Singapore–Cambridge General Certificate of Education Normal (Academic) Level (2024)

Mathematics Syllabus A (Syllabus 4045)

CONTENTS

INTRODUCTION

The syllabus is intended to provide students with fundamental mathematical knowledge and skills. The content is organised into three strands, namely, *Number and Algebra*, *Geometry and Measurement*, and *Statistics and Probability*. Besides conceptual understanding and skill proficiency explicated in the content strands, important mathematical processes such as reasoning, communication and application (including the use of models) are also emphasised and assessed.

AIMS

The N(A)-Level Mathematics syllabus aims to enable all students to:

- acquire mathematical concepts and skills for continuous learning in mathematics and to support learning in other subjects
- develop thinking, reasoning, communication, application and metacognitive skills through a mathematical approach to problem-solving
- connect ideas within mathematics, and between mathematics and other subjects, through applications of mathematics
- build confidence and foster interest in mathematics.

ASSESSMENT OBJECTIVES

The assessment will test candidates' abilities to:

AO1 Use and apply standard techniques

- recall and use facts, terminology and notation
- read and use information directly from tables, graphs, diagrams and texts
- carry out routine mathematical procedures

AO2 Solve problems in a variety of contexts

- interpret information to identify the relevant mathematics concept, rule or formula to use
- translate information from one form to another
- make and use connections across topics/subtopics
- formulate problems into mathematical terms
- analyse and select relevant information and apply appropriate mathematical techniques to solve problems
- interpret results in the context of a given problem

AO3 Reason and communicate mathematically

- justify mathematical statements
- provide explanation in the context of a given problem
- write mathematical arguments

Approximate weightings for the assessment objectives are as follows:

AO1	60%
AO2	30%
AO3	10%

SCHEME OF ASSESSMENT

Paper	Duration	Description	Marks	Weighting
Paper 1	2 hours	There will be about 23 short answer questions. Candidates are required to answer **all** questions.	70	50%
Paper 2	2 hours	Section A: There will be 9 – 10 questions of varying marks and lengths. The last question in this section will focus specifically on applying mathematics to a real-world scenario. Candidates are required to answer **all** questions. Section B: There will be 2 questions of which candidates will be required to answer **only one**. • The questions in this section will be based on the underlined content and there will be one question from the ‘Geometry and Measurement’ strand and one from the ‘Statistics and Probability’ strand. • Each question carries the same number of marks, that is, either 7 or 8 marks.	70	50%

NOTES

1. Omission of essential working will result in loss of marks.

2. Relevant mathematical formulae will be provided for candidates.

3. Candidates should also have geometrical instruments with them for both papers.

4. Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question. In questions which explicitly require an answer to be shown to be correct to a specific accuracy, the answer must be first shown to a higher degree of accuracy.

5. SI units will be used in questions involving mass and measures.

   Both the 12-hour and 24-hour clock may be used for quoting times of the day. In the 24-hour clock, for example, 3.15 a.m. will be denoted by 03 15; 3.15 p.m. by 15 15.

6. Candidates are expected to be familiar with the solidus notation for the expression of compound units, e.g. 5 cm/s for 5 centimetres per second, 13.6 g/cm^3 for 13.6 grams per cubic centimetre.

7. Unless the question requires the answer in terms of π, the calculator value for π or $\pi = 3.142$ should be used.

8. Spaces will be provided in each question paper for working and answers.

PROBLEMS IN REAL WORLD CONTEXTS

Notwithstanding the presentation of the topics in 3 separate strands in the syllabus document, it is envisaged that some examination questions (including the extended problem involving real-world contexts at the end of Section A of Paper 2) may integrate ideas from more than one topic.

Problems in real-world contexts may be based on contexts:

- In everyday life (including travel/excursion plans, transport schedules, sports and games, recipes, floor plans, navigation etc.)
- Involving personal and household finance (including simple and compound interest, taxation, instalments, utilities bills, money exchange, etc.)

These problems may also require:

- Interpreting and analysing data from tables and graphs, including distance-time and speed-time graphs
- Interpreting the solution in the context of the problem.

USE OF CALCULATORS

An approved calculator may be used in **both** Paper 1 and Paper 2.

SUBJECT CONTENT

Certain parts of the syllabus have been underlined and will only be tested in Section B of Paper 2.

No.	Topic/Sub-topics	Content
NUMBER AND ALGEBRA		
N1	**Numbers and their operations**	• primes and prime factorisation • finding highest common factor (HCF) and lowest common multiple (LCM), squares, cubes, square roots and cube roots by prime factorisation • negative numbers, integers, rational numbers, real numbers and their four operations • calculations with calculator • representation and ordering of numbers on the number line • use of $<$, $>$, $\leqslant$, $\geqslant$ • approximation and estimation (including rounding off numbers to a required number of decimal places or significant figures and estimating the results of computation) • use of standard form $A \times 10^n$, where n is an integer, and $1 \leqslant A < 10$ • positive, negative, zero and fractional indices • laws of indices
N2	**Ratio and proportion**	• comparison between two or more quantities by ratio • relationship between ratio and fraction • dividing a quantity in a given ratio • ratios involving rational numbers • equivalent ratios • writing a ratio in its simplest form • map scales (distance and area) • direct and inverse proportion
N3	**Percentage**	• expressing percentage as a fraction or decimal • expressing one quantity as a percentage of another • comparing two quantities by percentage • percentages greater than 100% • increasing/decreasing a quantity by a given percentage • finding percentage increase/decrease • reverse percentages
N4	**Rate and speed**	• relationships between distance, time and speed • writing speed in different units (e.g. km/h, m/min, m/s and cm/s) • calculation of speed, distance or time given the other two quantities • average rate and average speed • conversion of units (e.g. km/h to m/s)

N5	**Algebraic expressions and formulae**	• using letters to represent numbers • interpreting notations: ∗ ab as $a \times b$ ∗ $\frac{a}{b}$ as $a \div b$ or $a \times \frac{1}{b}$ ∗ a^2 as $a \times a$, a^3 as $a \times a \times a$, a^2b as $a \times a \times b$, … ∗ $3y$ as $y + y + y$ or $3 \times y$ ∗ $3(x + y)$ as $3 \times (x + y)$ ∗ $\frac{3+y}{5}$ as $(3 + y) \div 5$ or $\frac{1}{5} \times (3 + y)$ • evaluation of algebraic expressions and formulae • translation of simple real-world situations into algebraic expressions • recognising and representing patterns/relationships by finding an algebraic expression for the nth term • addition and subtraction of linear expressions • simplification of linear expressions such as: ∗ $-2(3x - 5) + 4x$ ∗ $\frac{2x}{3} - \frac{3(x-5)}{2}$ • use brackets and extract common factors • factorisation of linear expressions of the form $ax + bx + kay + kby$ • expansion of the product of algebraic expressions • changing the subject of a formula • finding the value of an unknown quantity in a given formula • use of: ∗ $(a + b)^2 = a^2 + 2ab + b^2$ ∗ $(a - b)^2 = a^2 - 2ab + b^2$ ∗ $a^2 - b^2 = (a + b)(a - b)$ • factorisation of quadratic expressions $ax^2 + bx + c$ • multiplication and division of simple algebraic fractions such as: ∗ $\left(\frac{3a}{4b^2}\right)\left(\frac{5ab}{3}\right)$ ∗ $\frac{3a}{4} \div \frac{9a^2}{10}$ • addition and subtraction of algebraic fractions with linear or quadratic denominator such as: ∗ $\frac{1}{x-2} + \frac{2}{x-3}$ ∗ $\frac{1}{x^2-9} + \frac{2}{x-3}$ ∗ $\frac{1}{x-3} + \frac{2}{(x-3)^2}$

No.	Topic/Sub-topics	Content

N6	**Functions and graphs**	• Cartesian coordinates in two dimensions
		• graph of a set of ordered pairs as a representation of a relationship between two variables
		• linear functions ($y = ax + b$) and quadratic functions ($y = ax^2 + bx + c$)
		• graphs of linear functions
		• the gradient of a linear graph as the ratio of the vertical change to the horizontal change (positive and negative gradients)
		• graphs of quadratic functions and their properties:
		  * positive or negative coefficient of x^2
		  * maximum and minimum points
		  * symmetry
		• graphs of power functions of the form $y = ax^n$, where $n = -2, -1, 0, 1, 2, 3$, and simple sums of not more than three of these
		• graphs of exponential functions $y = ka^x$, where a is a positive integer
		• estimation of the gradient of a curve by drawing a tangent

N7	**Equations and inequalities**	• solving linear equations in one variable
		• solving simple fractional equations that can be reduced to linear equations such as:
		  * $\frac{x}{3} + \frac{x-2}{4} = 3$
		  * $\frac{3}{x-2} = 6$
		• solving simultaneous linear equations in two variables by
		  * substitution and elimination methods
		  * graphical method
		• solving quadratic equations in one variable by
		  * factorisation
		  * use of formula
		  * completing the square for $y = x^2 + px + q$
		  * graphical method
		• solving fractional equations that can be reduced to quadratic equations such as:
		  * $\frac{6}{x+4} = x + 3$
		  * $\frac{1}{x-2} + \frac{2}{x-3} = 5$
		• formulating equations to solve problems
		• solving simple inequalities in the form $ax \leqslant b$, $ax \geqslant b$, $ax < b$ and $ax > b$ where a and b are integers

No.	Topic/Sub-topics	Content
GEOMETRY AND MEASUREMENT		
G1	**Angles, triangles and polygons**	• right, acute, obtuse and reflex angles • vertically opposite angles, angles on a straight line and angles at a point • angles formed by two parallel lines and a transversal: corresponding angles, alternate angles, interior angles • properties of triangles, special quadrilaterals and regular polygons (pentagon, hexagon, octagon and decagon) including symmetry properties • classifying special quadrilaterals on the basis of their properties • angle sum of interior and exterior angles of any convex polygon • construction of simple geometrical figures from given data using compasses, ruler, set squares and protractor, where appropriate

G2	**Congruence and similarity**	• congruent and similar figures
		• properties of similar triangles and polygons:
		  * corresponding angles are equal
		  * corresponding sides are proportional
		• enlargement and reduction of a plane figure
		• scale drawings
		• properties and construction of perpendicular bisectors of line segments and angle bisectors
		• solving simple problems involving similarity and congruence

G3	**Properties of circles**	• symmetry properties of circles:
		  * equal chords are equidistant from the centre
		  * the perpendicular bisector of a chord passes through the centre
		  * tangents from an external point are equal in length
		  * the line joining an external point to the centre of the circle bisects the angle between the tangents
		• angle properties of circles:
		  * angle in a semicircle is a right angle
		  * angle between tangent and radius of a circle is a right angle
		  * angle at the centre is twice the angle at the circumference
		  * angles in the same segment are equal
		  * angles in opposite segments are supplementary

G4	**Pythagoras' theorem and trigonometry**	• use of Pythagoras' theorem • determining whether a triangle is right-angled given the lengths of three sides • use of trigonometric ratios (sine, cosine and tangent) of acute angles to calculate unknown sides and angles in right-angled triangles • extending sine and cosine to obtuse angles • use of the formula $\frac{1}{2}ab\sin C$ for the area of a triangle • use of sine rule and cosine rule for any triangle • problems in two and three dimensions including those involving angles of elevation and depression and bearings

No.	Topic/Sub-topics	Content
G5	**Mensuration**	• area of parallelogram and trapezium • problems involving perimeter and area of composite plane figures • volume and surface area of cube, cuboid, prism, cylinder, pyramid, cone and sphere • conversion between cm^2 and m^2, and between cm^3 and m^3 • problems involving volume and surface area of composite solids • arc length as fraction of the circumference and sector area as fraction of the area of a circle • area of a segment • <u>use of radian measure of angle (including conversion between radians and degrees)</u> • <u>problems involving the arc length, sector area of a circle and area of a segment</u>
G6	**Coordinate geometry**	• finding the gradient of a straight line given the coordinates of two points on it • finding the length of a line segment given the coordinates of its end points • interpreting and finding the equation of a straight line graph in the form $y = mx + c$ • geometric problems involving the use of coordinates

No.	Topic/Sub-topics	Content
STATISTICS AND PROBABILITY		

S1	**Data handling and analysis**	• simple concepts in collecting, classifying and tabulating data
		• analysis and interpretation of:
		  * tables
		  * bar graphs
		  * pictograms
		  * line graphs
		  * pie charts
		  * dot diagrams
		  * histograms with equal class intervals
		  * stem-and-leaf diagrams
		  * cumulative frequency diagrams
		  * box-and-whisker plots
		• purposes and uses, advantages and disadvantages of the different forms of statistical representations
		• drawing simple inference from statistical diagrams
		• explaining why a given statistical diagram leads to misinterpretation of data
		• mean, mode and median as measures of central tendency for a set of data
		• purposes and use of mean, mode and median
		• calculation of the mean for grouped data
		• quartiles and percentiles
		• range, interquartile range and standard deviation as measures of spread for a set of data
		• calculation of the standard deviation for a set of data (grouped and ungrouped)
		• using the mean and standard deviation to compare two sets of data

S2	**Probability**	• probability as a measure of chance • probability of single events (including listing all the possible outcomes in a simple chance situation to calculate the probability) • probability of simple combined events (including using possibility diagrams and tree diagrams, where appropriate) • addition and multiplication of probabilities (mutually exclusive events and independent events)

MATHEMATICAL FORMULAE

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

MATHEMATICAL NOTATION

The list which follows summarises the notation used in Cambridge's Mathematics examinations. Although primarily directed towards A-Level, the list also applies, where relevant, to examinations at all other levels.

1. *Set Notation*

$\in$	is an element of
$\notin$	is not an element of
$\{x_1, x_2, \ldots\}$	the set with elements $x_1, x_2, \ldots$
$\{x: \ldots\}$	the set of all x such that
$\mathrm{n}(A)$	the number of elements in set A
$\varnothing$	the empty set
$\mathscr{E}$	universal set
A'	the complement of the set A
$\mathbb{Z}$	the set of integers, $\{0, \pm1, \pm2, \pm3, \ldots\}$
$\mathbb{Z}^+$	the set of positive integers, $\{1, 2, 3, \ldots\}$
$\mathbb{Q}$	the set of rational numbers
$\mathbb{Q}^+$	the set of positive rational numbers, $\{x \in \mathbb{Q}: x > 0\}$
$\mathbb{Q}_0^+$	the set of positive rational numbers and zero, $\{x \in \mathbb{Q}: x \geqslant 0\}$
$\mathbb{R}$	the set of real numbers
$\mathbb{R}^+$	the set of positive real numbers, $\{x \in \mathbb{R}: x > 0\}$
$\mathbb{R}_0^+$	the set of positive real numbers and zero, $\{x \in \mathbb{R}: x \geqslant 0\}$
$\mathbb{R}^n$	the real n-tuples
$\mathbb{C}$	the set of complex numbers
$\subseteq$	is a subset of
$\subset$	is a proper subset of
$\nsubseteq$	is not a subset of
$\not\subset$	is not a proper subset of
$\cup$	union
$\cap$	intersection
$[a, b]$	the closed interval $\{x \in \mathbb{R}: a \leqslant x \leqslant b\}$
$[a, b)$	the interval $\{x \in \mathbb{R}: a \leqslant x < b\}$
$(a, b]$	the interval $\{x \in \mathbb{R}: a < x \leqslant b\}$
(a, b)	the open interval $\{x \in \mathbb{R}: a < x < b\}$

2. *Miscellaneous Symbols*

$=$	is equal to
$\neq$	is not equal to
$\equiv$	is identical to or is congruent to
$\approx$	is approximately equal to
$\propto$	is proportional to
$<$	is less than
$\leqslant$; $\ngtr$	is less than or equal to; is not greater than
$>$	is greater than
$\geqslant$; $\nless$	is greater than or equal to; is not less than
∞	infinity

3. *Operations*

$a + b$	a plus b
$a - b$	a minus b
$a \times b$, ab, $a.b$	a multiplied by b
$a \div b$, $\frac{a}{b}$, a/b	a divided by b
$a : b$	the ratio of a to b
$\sum_{i=1}^{n} a_i$	$a_1 + a_2 + ... + a_n$
$\sqrt{a}$	the positive square root of the real number a
$\lvert a \rvert$	the modulus of the real number a
$n!$	n factorial for $n \in \mathbb{Z}^+ \cup \{0\}$, $(0! = 1)$
$\binom{n}{r}$	the binomial coefficient $\frac{n!}{r!(n-r)!}$, for $n, r \in \mathbb{Z}^+ \cup \{0\}$, $0 \leqslant r \leqslant n$ $\frac{n(n-1)...(n-r+1)}{r!}$, for $n \in \mathbb{Q}$, $r \in \mathbb{Z}^+ \cup \{0\}$

4. *Functions*

f	the function f
$\mathrm{f}(x)$	the value of the function f at x
$\mathrm{f}: A \to B$	f is a function under which each element of set A has an image in set B
$\mathrm{f}: x \mapsto y$	the function f maps the element x to the element y
f^{-1}	the inverse of the function f
$\mathrm{g} \circ \mathrm{f}$, gf	the composite function of f and g which is defined by $(\mathrm{g} \circ \mathrm{f})(x)$ or $\mathrm{gf}(x) = \mathrm{g}(\mathrm{f}(x))$
$\lim\limits_{x \to a} \mathrm{f}(x)$	the limit of $\mathrm{f}(x)$ as x tends to a
Δx ; δx	an increment of x
$\dfrac{\mathrm{d}y}{\mathrm{d}x}$	the derivative of y with respect to x
$\dfrac{\mathrm{d}^n y}{\mathrm{d}x^n}$	the nth derivative of y with respect to x
$\mathrm{f}'(x), \mathrm{f}''(x), \ldots, \mathrm{f}^{(n)}(x)$	the first, second, … nth derivatives of $\mathrm{f}(x)$ with respect to x
$\int y \, \mathrm{d}x$	indefinite integral of y with respect to x
$\int_a^b y \, \mathrm{d}x$	the definite integral of y with respect to x for values of x between a and b
$\dot{x}, \ddot{x}, \ldots$	the first, second, …derivatives of x with respect to time

5. *Exponential and Logarithmic Functions*

e	base of natural logarithms
e^x, $\exp x$	exponential function of x
$\log_a x$	logarithm to the base a of x
$\ln x$	natural logarithm of x
$\lg x$	logarithm of x to base 10

6. *Circular Functions and Relations*

sin, cos, tan, cosec, sec, cot	the circular functions
$\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$ cosec^{-1}, $\sec^{-1}$, $\cot^{-1}$	the inverse circular functions

7. *Complex Numbers*

i	the square root of -1
z	a complex number, $z = x + \mathrm{i}y$ $= r(\cos\theta + \mathrm{i}\sin\theta),\ r \in \mathbb{R}_0^+$ $= r\mathrm{e}^{\mathrm{i}\theta},\ r \in \mathbb{R}_0^+$
$\mathrm{Re}\ z$	the real part of z, $\mathrm{Re}\ (x + \mathrm{i}y) = x$
$\mathrm{Im}\ z$	the imaginary part of z, $\mathrm{Im}\ (x + \mathrm{i}y) = y$
$\lvert z \rvert$	the modulus of z, $\lvert x + \mathrm{i}y \rvert = \sqrt{x^2 + y^2}$, $\lvert r(\cos\theta + \mathrm{i}\sin\theta) \rvert = r$
$\arg z$	the argument of z, $\arg(r(\cos\theta + \mathrm{i}\sin\theta)) = \theta$, $-\pi < \theta \leqslant \pi$
z^*	the complex conjugate of z, $(x + \mathrm{i}y)^* = x - \mathrm{i}y$

8. *Matrices*

$\mathbf{M}$	a matrix $\mathbf{M}$
$\mathbf{M}^{-1}$	the inverse of the square matrix $\mathbf{M}$
$\mathbf{M}^{\mathrm{T}}$	the transpose of the matrix $\mathbf{M}$
det $\mathbf{M}$	the determinant of the square matrix $\mathbf{M}$

9. *Vectors*

$\mathbf{a}$	the vector $\mathbf{a}$
$\overrightarrow{AB}$	the vector represented in magnitude and direction by the directed line segment AB
$\hat{\mathbf{a}}$	a unit vector in the direction of the vector $\mathbf{a}$
$\mathbf{i}, \mathbf{j}, \mathbf{k}$	unit vectors in the directions of the Cartesian coordinate axes
$\lvert \mathbf{a} \rvert$	the magnitude of $\mathbf{a}$
$\lvert \overrightarrow{AB} \rvert$	the magnitude of $\overrightarrow{AB}$
$\mathbf{a}\cdot\mathbf{b}$	the scalar product of $\mathbf{a}$ and $\mathbf{b}$
$\mathbf{a}\times\mathbf{b}$	the vector product of $\mathbf{a}$ and $\mathbf{b}$

10. *Probability and Statistics*

A, B, C, etc.	events
$A \cup B$	union of events A and B
$A \cap B$	intersection of the events A and B
$\mathrm{P}(A)$	probability of the event A
A'	complement of the event A, the event 'not A'
$\mathrm{P}(A \mid B)$	probability of the event A given the event B
X, Y, R, etc.	random variables
x, y, r, etc.	value of the random variables X, Y, R, etc.
$x_1, x_2, \ldots$	observations
$f_1, f_2, \ldots$	frequencies with which the observations, $x_1, x_2, \ldots$occur
$\mathrm{p}(x)$	the value of the probability function $\mathrm{P}(X = x)$ of the discrete random variable X
$p_1, p_2, \ldots$	probabilities of the values $x_1, x_2, \ldots$of the discrete random variable X
$\mathrm{f}(x)$, $\mathrm{g}(x)\ldots$	the value of the probability density function of the continuous random variable X
$\mathrm{F}(x)$, $\mathrm{G}(x)\ldots$	the value of the (cumulative) distribution function $\mathrm{P}(X \leqslant x)$ of the random variable X
$\mathrm{E}(X)$	expectation of the random variable X
$\mathrm{E}[\mathrm{g}(X)]$	expectation of $\mathrm{g}(X)$
$\mathrm{Var}(X)$	variance of the random variable X
$\mathrm{B}(n, p)$	binomial distribution, parameters n and p
$\mathrm{Po}(\mu)$	Poisson distribution, mean μ
$\mathrm{N}(\mu, \sigma^2)$	normal distribution, mean μ and variance σ^2
μ	population mean
σ^2	population variance
σ	population standard deviation
$\bar{x}$	sample mean
s^2	unbiased estimate of population variance from a sample, $s^2 = \frac{1}{n-1}\Sigma(x - \bar{x})^2$
φ	probability density function of the standardised normal variable with distribution $\mathrm{N}(0, 1)$
Φ	corresponding cumulative distribution function
ρ	linear product-moment correlation coefficient for a population
r	linear product-moment correlation coefficient for a sample